Transmission of unknown variable length quantum strings

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Abstract. A variable length quantum string is a quantum string which is in a superposition of different lengths. When a quantum string is compressed using a lossless quantum code, the compressed string is variable length. Therefore in order to losslessly code a quantum string, it is necessary to be able to transmit a variable length quantum string. Previous lossless quantum coding schemes have been lossy because the strings being transmitted have been measured in order to determine how many qubits to transmit.

We show how unknown variable length quantum strings can be transmitted through quantum channels without using any

Keywords: Transmission, indeterminant length, variable length, lossless quantum coding, lossless quantum compression

1 Introduction

Transmission is the movement of information between two distinct systems. Transmission is often used to transmit files between two computers. Many of the problems in information theory are phrased in terms of two parties communicating over a channel. Transmission is therefore central to information theory.

A bit is a variable which can take the value 0 or 1. Information is represented internally on a classical computer as strings (or sequences) of bits. The number of bits in a string is the length of the string. Information on quantum computers can be represented as a superposition of classical strings. If a quantum string is a superposition of classical strings of the same length, then that quantum string is a fixed length string. If a quantum string is a superposition of classical strings of differing lengths, then that quantum string is a variable-length quantum string. For example, $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ is a fixed length quantum string with length 2 whereas $\frac{1}{\sqrt{2}}(|01\rangle + |100\rangle)$ is a variable-length quantum string.

A variable length quantum string is in a superposition of distinct lengths, so the length of an unknown quantum string cannot be accurately measured (without error). If any measurement is made on an unknown variable length quantum string which gains information about the lengths in the superposition, then the unknown quantum string is disturbed (a loss in fidelity).

The aim of lossless coding is to transmit a possibly unknown quantum state between two parties by transmitting the fewest number of qubits on average with zero-error probability and with perfect fidelity. Lossless coding first compresses the state, transmits the state and the second party decompresses the state. If the set of states to be coded are not orthogonal, then the output of the compression part of the algorithm is a variable length quantum string. Previous papers on lossless quantum coding [2, 4, 1, 3] have stated that transmission of unknown variable length quantum strings is not possible because of additional assumptions. We will present a simple protocol for transmitting variable length quantum strings and discuss its significance.

2 Transmission of variable length quantum strings

We shall illustrate our transmission protocol with an example. Two parties, Alice and Bob, are attempting to communicate over a quantum channel. Alice is attempting to send Bob some unknown quantum string

$$|\psi\rangle = \alpha|0\rangle + \beta|10\rangle + \gamma|110\rangle + \delta|111\rangle$$

which is a variable length superposition of the pseudo-classical states $|0\rangle$, $|10\rangle$, $|110\rangle$ and $|111\rangle$.

A string y is a prefix of a string x, if there exists z such that $yz = x$. For example, the non-empty prefixes of 110 are 1, 11 and 110. The strings in the superposition $|\psi\rangle$ form a set of prefix code words because no string is a prefix of any other string. Therefore the end of each string in the superposition is immediately recognizable. We can therefore assume that $|\psi\rangle$ is zero-padded on Alice's computer; each string in the superposition is followed by an array of $|0\rangle$'s so that when $|\psi\rangle$ is zero-padded it belongs to a Hilbert space of fixed dimension. The first three qubits of Alice's computer are therefore:

$$\alpha|000\rangle + \beta|100\rangle + \gamma|110\rangle + \delta|111\rangle$$

Our transmission protocol is similar to the one contained in [4], however we also assume that Bob's computer is initialized with an infinite array of $|0\rangle$'s. We can write the joint state $H_A \otimes H_B$ of the first three qubits of Alice's and Bob's computer with Hilbert spaces H_A and H_B respectively before the transmission as:

$$(\alpha|000\rangle + \beta|100\rangle + \gamma|110\rangle + \delta|111\rangle) \otimes |000\rangle$$

After the transmission, the joint state can be written as:

$$|000\rangle \otimes (\alpha|000\rangle + \beta|100\rangle + \gamma|110\rangle + \delta|111\rangle)$$

Because the state transmitted is the superposition of a prefix code, the transmission can be accomplished unitarily. There is no entanglement between Alice and Bob's computers after the transmission. In general any quantum prefix code can be transmitted.

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2.1 How many qubits are transmitted?

Classically, if Alice sends Bob a file which is terminated by an end marker (a form of prefix coding) then in theory, the length of the string that Alice sends to Bob is infinite. We say that Alice sends Bob a finite number of bits because the number of bits that are transferred through the channel is dependent solely on the state of Alice's computer. It is the same in the example above, the number of qubits which actually flow through the channel connecting Alice and Bob's computer is dependent only on the string contained in Alice's computer. So if γ and δ are both 0 then only at most two qubits are transmitted.

3 Applications of variable length transmission and future work

Variable length transmission is an application in itself. A quantum string is a superposition of classical strings, and in general a quantum string is variable length. So variable length transmission is necessary for the transmission of general quantum states.

Variable length transmission is a requirement for lossless quantum coding of unknown non-orthogonal states. The number of qubits transmitted is not an observable, however if the channel being used can transmit a limited number of qubits per unit time, then quantum lossless coding can be used to minimize the expected time for a quantum string to be transmitted.

Universal coding is a generalization of lossless coding which asymptotically achieves the optimal rate of coding (expected number of bits to be transmitted) using fewer assumptions about the knowledge of the state to be coded than standard lossless coding. Transmission of variable length codes is necessary for lossless universal coding because the basis for compression is unknown. We will show that lossless universal quantum coding is possible.

Kolmogorov complexity is a further generalization of lossless coding. As a generalization of lossless coding, Kolmogorov is more widely applicable as a theoretical tool, though not practically viable because the Kolmogorov complexity of a string is uncomputable due to the Halting problem. The Kolmogorov complexity of a string is defined as the shortest description of that string in a fixed universal language such as C, pure LISP or the language of some fixed universal Turing machine. Quantum Kolmogorov complexity has been described as the shortest description of a quantum string on a quantum Turing machine. The shortest description of a string can be seen as the maximal coding rate of that string. The present definition of Kolmogorov complexity does not allow variable length inputs. If variable length quantum strings are used for lossless quantum coding, then since Kolmogorov complexity is a generalization of lossless coding, one would expect variable length descriptions to be allowed. We plan to extend the definition of quantum Kolmogorov complexity to variable length quantum strings.

4 Acknowledgements

Much of this work was carried out at the Quantum Optics and Laser Science group at Imperial College, London. We would like to thank the group for their hospitality. We would also like to thank Vlatko Vedral, Luke Rallan, Garry Bowen, Rod Moore and Elham Kashefi for useful discussion.

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