NMR experiments on in-place addition circuits using quantum Fourier transformation

Yoshitaka Tsurukame[1 2 *] Masahiro Kitagawa[1 2 †]

[1] *Graduate School of Engineering Science, Osaka University*
1-3 Machikaneyama-cho, Toyonaka, Osaka 560-8531 JAPAN
[2] *CREST, Japan Science and Technology Corporation*

Abstract. Full adder usually requires carry bits. However, the in-place addition based on the quantum Fourier transformation (QFT) does not require any extra bit, which was proposed by A. Fijany in 1998 and reported by T. G. Draper. It is very useful in the present situation where only a limited number of qubits are available. Moreover, it is essential when no clear bit is available as in the case of initialization circuit of NMR quantum computer. In this paper, we have experimentally demonstrated the QFT-based addition for the first time, using NMR quantum computer. We have demonstrated the quantum parallelism by performing additions to superposition.

Keywords: addition, quantum Fourier transformation, carry bit

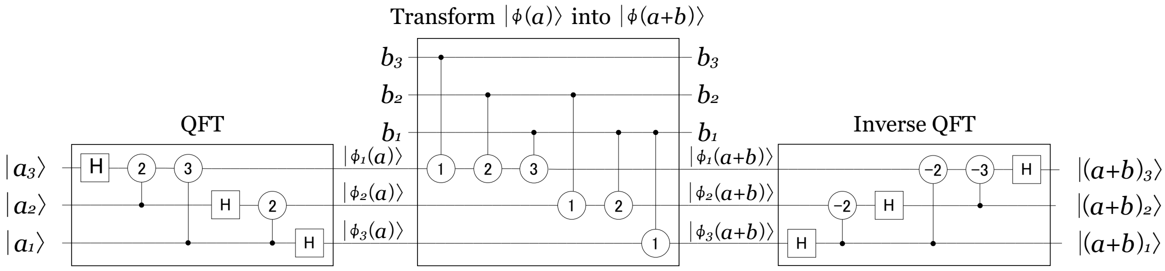


Figure 1: The addition circuit using quantum Fourier transformation

1 Full adder without carry bits

The N-qubit addition based on classical addition algorithm requires another $(N-1)$ carry qubits, which must be initialized to $|0\rangle$. The usage of extra qubits is not desirable in the present situation where we have only a limited number of qubits. Moreover, it can not be used in the initialization circuit where clear carry qubits are not available in the first place [1] .

This problem can be solved by addition algorithm using quantum Fourier transformation (QFT) [2, 3] which was proposed by Amir Fijany *et al.* in 1998 [4] and was reported by Thomas G. Draper [5] . However, the algorithm has not been tested experimentally. In this paper, we demonstrate this algorithm for the first time using nuclear magnetic resonance (NMR).

2 The addition circuit using QFT

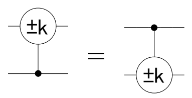
In the classical Fourier transformation, the shifting property,

$$\mathcal{F}[f(x+a)] = e^{iat}\mathcal{F}[f(x)], \qquad (1)$$

is well-known. Accordingly, we can perform addition by using QFT and phase rotation [4, 5] .

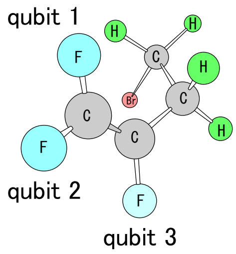
In the case of 3-qubit, the addition circuit can be composed as shown in Fig. 1 where 2-qubit controlled gates are defined in Fig. 2. We represent n-qubit QFT as

$$|\phi(x)\rangle = \frac{1}{2^{\frac{n}{2}}} \sum_{k=0}^{2^n-1} \exp(\frac{2\pi i x k}{2^n})|k\rangle. \qquad (2)$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \exp(\pm\frac{2\pi i}{2^k}) \end{pmatrix}$$

Figure 2: The conditional rotation gate ($k = 1, 2, 3, \cdots$)



(a) Structure

Chemical shifts	qubit 1	-6228.77Hz
	qubit 2	2468.58Hz
	qubit 3	-31300.95Hz
Spin-spin coupling constants	J_{12}	86.68Hz
	J_{13}	-113.51Hz
	J_{23}	32.52Hz

(b) The parameter about the qubits

Figure 3: $^{12}C_4H_4BrF_3$ as 3-qubit molecule

When QFT is performed at the beginning of the circuit, an augend $a = a_3 2^2 + a_2 2^1 + a_1 2^0$ becomes $\phi(a)$. The k-th bit is given by,

$$|\phi_k(a)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + \exp(\frac{2\pi i a}{2^k})|1\rangle). \qquad (3)$$

Denoting $e(t) = e^{2\pi i t}$, it is possible to express the phase factor as $e(a/2^k) = e(0.a_k \cdots a_1)$. Therefore, $|\phi_k(a)\rangle$ contains the lowest k digits of the binary a in its phase.

Next, phase shifts controlled by an addend $b = b_3 2^2 + b_2 2^1 + b_1 2^0$ are performed at the center of the circuit. $\phi_3(a)$ evolves by the subsequent applications of phase shifts as

*tsurukame@qc.ee.es.osaka-u.ac.jp
†kitagawa@ee.es.osaka-u.ac.jp

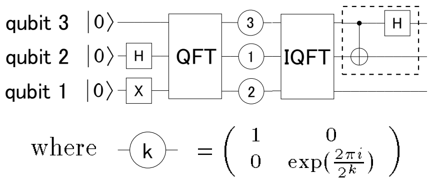
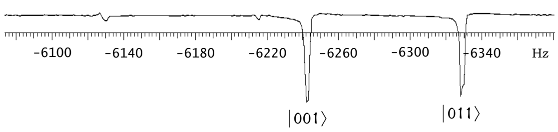
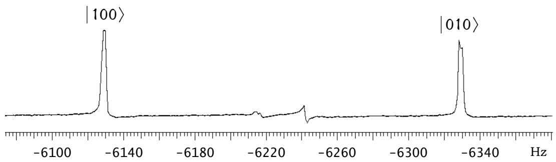


Figure 4: The 3-qubit QFT-based addition circuit and the disentangling circuit (dashed box) to examine the coherence of the addition



(a) The superposition state $|001\rangle + |011\rangle$



(b) After adding 001 to $|001\rangle + |011\rangle$

Figure 5: The spectra which were observed in each step to add 001 to $|001\rangle + |011\rangle$

$$
\begin{aligned}
|\phi_3(a)\rangle &\longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + e(0.a_3a_2a_1 + 0.b_3)|1\rangle) \\
&\longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + e(0.a_3a_2a_1 + 0.b_3b_2)|1\rangle) \\
&\longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + e(0.a_3a_2a_1 + 0.b_3b_2b_1)|1\rangle) \\
&= |\phi_3(a+b)\rangle, \qquad (4)
\end{aligned}
$$

and similarly, $|\phi_2(a)\rangle \longrightarrow |\phi_2(a+b)\rangle$, $|\phi_1(a)\rangle \longrightarrow |\phi_1(a+b)\rangle$. As a whole $|\phi(a)\rangle$ turns into $|\phi(a+b)\rangle$.

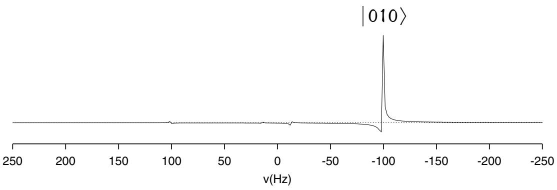
Finally, inverse QFT (IQFT) is performed and $\phi(a+b)$ becomes $a+b$. In this way, addition is performd using QFT and phase shifts without requiring carry qubits.

3 Experiments using NMR

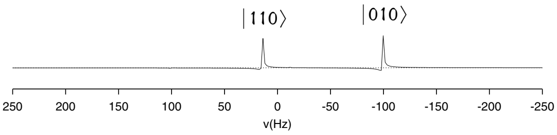
We have used the molecule shown in Fig. 3 to experimentally implement the quantum circuit in Fig. 1. We have used three ^{19}F nuclear spins as qubits by decoupling protons. Since the initial state is thermal equilibrium state in the NMR quantum computer [6, 7] , initialization is performed by the simplified exhaustive averaging [8] .

To demonstrate the parallel computation, we have performed the sum with the augend input in the superposition state $|001\rangle + |011\rangle$ (normalization factor omitted) and the scalar addend $010_2 = 2_{10}$. We observe the NMR spectra of the qubit 1. Fig. 5(a) shows the spectrum of the input augend state. Fig. 5(b) shows the spectrum of the output state in the position $|010\rangle$ and $|100\rangle$, which correspond to $|1+1\rangle$ and $|3+1\rangle$ respectively.

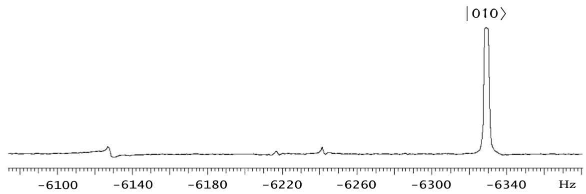
However, the spectrum of Fig. 5(b) alone can not distinguish superposition and mixture of $|010\rangle$ and $|100\rangle$. To examine the output state further, we have applied disentangling operation to the qubits 3 and 2. The whole circuit diagram including addition part is shown in Fig.



(a) The simulations for the superposition



(b) The simulations for the mixed state



(c) The result of the experiment

Figure 6: The simulations and the result of the experiment of the circuit in Fig. 4

4. The result of the simulation for the superposition is shown in Fig. 6(a), and that for the mixture is in Fig. 6(b). The experimental spectrum has clearly appeared in the position $|010\rangle$ as shown in Fig. 6(c) and suggests that Fig. 5(b) is in the superposition. That is, the parallel computation of $1+1=2$ and $3+1=4$,

$$|001\rangle + |011\rangle \xrightarrow{\text{add } 001} |010\rangle + |100\rangle, \qquad (5)$$

has been successfully performed.

We have also demonstrated the cascaded applications of QFT-based addition and the addition to the entangled augend, which have been omitted from this abstract.

4 Conclusion

By the NMR experiments, we have demonstrated that the QFT-based addition circuit operates properly for quantum parallel computation.

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