NMR quantum information processing without pseudo-pure state

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Abstract. An experimental approach is proposed for superdense coding to test the existence of entanglement in NMR. This approach does not require any preparation of pseudo-pure state. The result suggests either that the quantitative threshold must be introduced to the experiment to assure the existence of entanglement or that the mathematical criterion for mixed state entanglement is unphysical.

Keywords: NMR, entanglement criterion, superdense coding

1 Introduction

Entanglement is a fundamental prerequisite for non-local algorithms such as quantum teleportation [1] and dense coding[2]. The seemingly successful demonstration of bulk ensemble NMR quantum teleportation by Nielsen *et al.* [3] has brought about the confusion on the existence of entanglement, because it uses, as most NMR experiments do, a pseudo pure state created from a highly mixed state, from which no entanglement can arise according to the mathematical proof by Braunstein *et al.* [4].

The controversy over NMR experiments focuses on the separability of the density matrix evolved from the pseudo-pure state due to extremely low nuclear spin polarization at room temperature. However, the initial spin polarization can be enhanced to the extent where entanglement can exist according to the theoretical criterion by lowering the temperature or using the dynamic nuclear polarization. Therefore, NMR experiment *with entanglement* is possible.

On the other hand, pseudo-pure state might have conceptual problem in that different experiments are summed up [5] or in-homogenous experiments are integrated [6] and described by a single density operator. Therefore some loss of information is inherent in the pseudo-pure state and it might be inappropriate to rely on the scheme when discussing such a dedicate issue as the existence of entanglement. The motivation of this paper is to examine the existence of entanglement in bulk ensemble NMR experiments without introducing pseudo-pure state.

2 Superdense coding experiment

As the simplest example of non-local experiments, let us consider quantum superdense coding, whose circuit, shown in Fig. 1, requires only two qubits. Its goal is to transmit two classical bits of information from Alice to Bob by sending a single qubit. For the unitary transformation U, Alice chooses one of the four operations $\{I, \sigma_x, \sigma_y, \sigma_z\}$, and the result of the Bell measurement performed by Bob tells him Alice's decision.

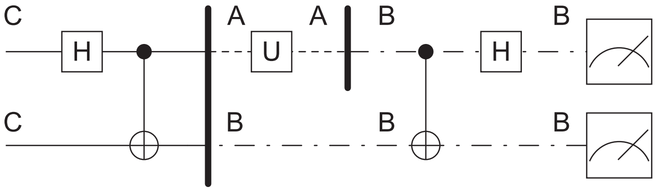

Figure 1: A quantum circuit for superdense coding. Bold vertical lines indicate the hand over of qubits, which correspond to the transmissions in the original superdense coding. The left part is Charles (C), who creates two-qubit entangled state (stroke lines) and sends one (dashed line) to the Alice (A) and the other (dashed-dotted line) to Bob (B). In the middle part, Alice selects one of the four unitary operations $\{I, \sigma_x, \sigma_y, \sigma_z\}$ for U, and sends her qubit to Bob. In the right part, Bob performs the Bell measurement by converting the basis into the computational basis and measuring each qubit. The result contains two-bit classical information, from which Bob can tell the choice of Alice on U.

We assume the initial state $\rho_{\text{ini}} = \rho_1 \otimes \rho_2$ with $\rho_1 = \rho_2 = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$, where p and $1-p$ are the probabilities for finding the quantum state of spin to be $|0\rangle$ and $|1\rangle$, respectively, and p is related to the spin polarization ϵ through $p = (1+\epsilon)/2$. At temperature T and in a static magnetic field B_0, thermal-equilibrium polarization is given by $\epsilon_{\text{eq}} = \tanh(\gamma\hbar B_0/2kT)$, where γ is the gyromagnetic ratio and k is the Boltzmann constant.

For the present case, the existence of entanglement, which is supposed to be essential for successful superdense coding, will depend on the initial spin polarization ϵ. From a mathematical separability criterion [7, 8], a threshold $\epsilon_{\text{th}} = \sqrt{2} - 1 \sim 0.41$ can be obtained for the initial spin polarization necessary to create entanglement between Alice and Bob. Considering the argument that the entanglement is essential in order for such a non-local experiment to be successful, the initial polarization ϵ ought to be greater than this threshold. Unfortunately, ϵ_{th} is five orders of magnitude larger than the thermal-equilibrium polarization available in conventional liquid state NMR experiments at room temperature. However it is possible to obtain higher polarization than ϵ_{th} in solid state even at moderate and conveniently-available

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temperature by means of dynamic nuclear polarization (DNP) [9, 10], by which we plan to realize solid-state NMR quantum computing with high polarization.

3 Discussions

The problem is that the signals corresponding to a successful superdense coding are observed even if there is no entanglement according to the mathematical criterion. There is no qualitative difference between the results of the NMR experiments performed above and below the threshold. The difference is only qualitative since the measured signal intensity is proportional to ϵ. This fact may raise discrepancy between the theoretical prognosis and the experimental observations.

If we accept the mathematical non-separability criterion for the existence of entanglement, it follows that an experimental criterion has to be introduced for the existence of entanglement based on the signal intensity in order to keep consistency. The signal intensity for i-th qubit is obtained from the reduced density operator $\rho_i^{\mathrm{out}} = \mathrm{Tr}_j \rho_{ij}^{\mathrm{out}}$. In the case of $U = I$, the quantum circuit in Fig. 1 does nothing and obviously the input state appears at the output as $\rho_i^{\mathrm{out}} = \rho_i$ $(i = 1, 2)$. The signal intensity normalized by that of the fully polarized case represents ϵ. Therefore the normalized signal intensity which corresponds to the mathematical threshold is given by ϵ_{th}. Other choices of U may change only the sign of the signals and not their strength. Therefore the threshold is independent from the choice of U.

If we adopt the threshold, we have to discard the experimental results with signal strength below ϵ_{th}. Although we will be able to perform experiments above the threshold in the near future, still the question remains if the experiments below and above the threshold are fundamentally different in physics. If this is not the case, the mathematical criterion for mixed state entanglement based on the separability of the density operator is unphysical.

4 Conclusions

We have not resolved the issue concerning the exsistence of entanglement in NMR experimients. However, we have clarified the issue by avoiding the conceptually problematic psudo-pure state in constructing the superdense coding experiment. The source of the discrepancy between the mathematical criterion and the physical observations concerning the existence of entanglement might reside deep in the nature of mixed state.

References

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels," *Phys. Rev. Lett.* **70**, pp. 1895–1899 (1993).

[2] C. H. Bennett and S. J. Wiesner, "Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states," *Phys. Rev. Lett.* **69**, pp. 2881–2884 (1992).

[3] M. A. Nielsen, E. Knill, and R. Laflamme, "Complete quantum teleportation using nuclear magnetic resonance," *Nature* **396**, pp. 52–55 (1998).

[4] S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, "Separability of Very Noisy Mixed States and Implications for NMR Quantum Computing", *Phys. Rev. Lett.* **83**, pp. 1054–1057 (1999).

[5] E. Knill, I. Chuang, and R. Laflamme, "Effective pure states for bulk quantum computation," *Phys. Rev. A*, **57**, pp. 3348–3363 (1998).

[6] D. G. Cory, A. F. Fahmy, and T. F. Havel, "Ensemble quantum computing by NMR spectroscopy," *Proc. Natl. Acad. Sci. USA*, **94**, pp. 1634–1639 (1997).

[7] A. Peres, "Separability Criterion for Density Matrices," *Phys. Rev. Lett.* **77**, pp. 1413–1415 (1996).

[8] M. Horodecki, P. Horodecki, and R. Horodecki, "Separability of mixed states: necessary and sufficient conditions," *Phys. Lett. A* **223**, pp. 1–8 (1996).

[9] K. Takeda, T. Takegoshi, and T. Terao, *Phys. Rev. B*, Submitted.

[10] K. Takeda, "Studies on dynamic nuclear polarization by photo-excited triplet electron spins," PhD Thesis (Dept. Chem., Grad. Sch. Sci., Kyoto University) (2003).