Acceleration of quantum algorithms using three-qubit gates

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Abstract. We consider a method to overcome decoherence through constructing quantum circuits using larger building blocks. Our strategy is to numerically find the control-parameter sequences which produce the desired gates. We demonstrate how to implement the famous three-qubit Toffoli, Fredkin, QFT and the phase-shift gates on the Josephson charge-qubit model. We also discuss the robustness of the gates obtained. Furthermore, we compare the length of the control-parameter sequences and find that the ability to directly implement three-qubit gates — instead of the elementary gate decomposition — considerably reduces the execution times of quantum algorithms.

Keywords: decoherence, Josephson charge qubit, multiqubit quantum gates, numerical optimization

1 Introduction

The standard approach to implementing a quantum algorithm is to build the unitary operations using the set of elementary gates [1] which are typically chosen to be the one-qubit unitary rotations and the CNOT gate. The resulting decomposition into elementary gates, the quantum circuit, mimics the operation principle of a classical digital computer. The elementary gate decomposition of a given quantum algorithm may require a large number of gates. However, the gate sequence can be shortened considerably by introducing more complicated two-, and three-qubit gates in addition to the elementary gates [2]. Thus, for a given decoherence time, the ability of implementing arbitrary three-qubit gates will enable the execution of much larger algorithms.

2 Josephson charge-qubit model

The Josephson charge qubit utilizes the number degree of freedom of Cooper pairs in a nanoscale Josephson-junction circuit. The states of the qubit correspond to either zero or one extra Cooper pair residing on the superconducting island. The charging energy of the qubit can be tuned with the help of an external gate voltage, whereas tunneling between the states is controlled with the help of an external magnetic flux. The Hamiltonian for the qubit register is [3]

$$H = \sum_{i}^{N}\left\{-\frac{1}{2}B_z^i\sigma_z^i - \frac{1}{2}B_x^i\sigma_x^i\right\} - \sum_{i,j}^{N,N} C_{ij}B_x^iB_x^j\sigma_y^i\otimes\sigma_y^j, \quad (1)$$

where the standard notation for Pauli matrices has been utilized. Here B_z^i and B_x^i are tunable parameters, which depend on the gate voltages and the enclosed magnetic fluxes.

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3 Numerical methods for finding multi-qubit gates

The temporal evolution of the quantum state of a Josephson-qubit register induces the unitary matrix

$$U_\gamma = \mathcal{T}\exp\left(-i\int_{\gamma(t)} H(\gamma(t))dt\right), \quad (2)$$

where $\mathcal{T}$ stands for the time-ordering operator. The integration is performed along the path $\gamma(t)$ in parameter space which describes the time evolution of the control parameters $\{B_x^j(t)\}$ and $\{B_z^j(t)\}$.

We will here consider a special class of paths $\gamma(t)$, which form polygons in the parameter space. Accordingly the fields, described by the parameters, vary in time at a piecewise constant speed.

We transform the problem of finding the desired unitary operator into an optimization task. Namely, any $\hat{U}$ can be found as zero of the error functional

$$f(\gamma) = \|\hat{U} - U_\gamma\|_F, \quad (3)$$

which is found by minimizing over all possible polygonal paths $\gamma(t)$. Above, the subscript F refers to the Frobenius trace norm for matrices.

Note that the polygon γ has to have enough vertices to parameterize the unitary group $SU(2^N)$. The minimization landscape is rough, see Fig. 1, and thus we apply the robust polytope algorithm for the minimization.

4 Results

We have applied the minimization procedure to the various two- and three-qubit gates and found that the error functional of Eq. (3) can be minimized in a reasonable time by using a parallel computer. We consider as an example the implementation of the controlled^2 phase-shift gate as well as the familiar Toffoli, Fredkin, and three-qubit QFT gates. The results are qualitatively similar for all gates. Figure 2 shows the found control parameter sequence $\gamma(t)$ which yields the desired gate at high accuracy.

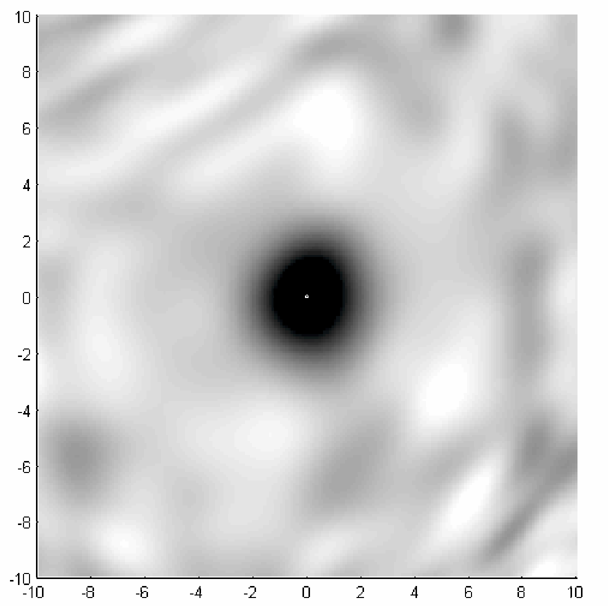

Figure 1: Error functional landscape (a planar cut) for controlled^2 phase-shift gate. The optimal minimum $3 \cdot 10^{-4}$ is situated in the center of the plane.

We found that the error functional grows linearly in the vicinity of the minimum, which implies that the parameter sequence found here may be robust.

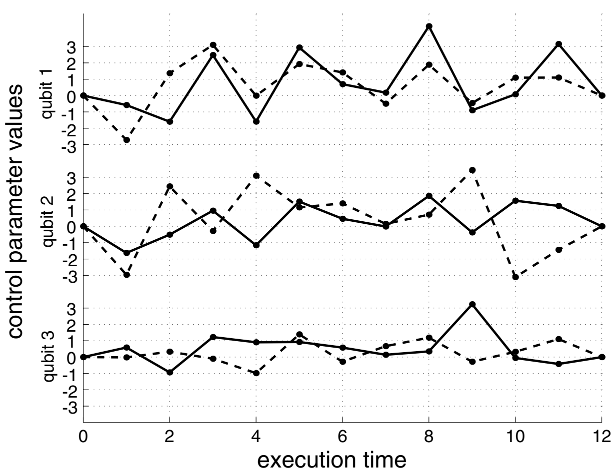

Figure 2: Control-parameter sequences for the controlled^2 phase-shift gate. Solid lines represent the field B_z^i while the dotted lines denote the field B_x^i, respectively.

In our scheme, the execution time of the quantum algorithm depends on the number of vertices of the parameter path. For the example cases, we compare the number of edges that are required to carry out a single three-qubit gate or using a sequence of elementary gates. We find that the implementation is improved if the three-qubit gate is utilized.

We conclude that more efficient implementations of quantum algorithms may be achieved if multiqubit gates are employed in general, instead of elementary quantum gates such as single-qubit and CNOT gates, as the building blocks. The price we pay is that we must carry out heavy computations on a classical computer but as a result we can definitely shorten the operation time and are hence able to fight decoherence.

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