Probabilistic Teleportation of multi-Particle d-Level Quantum State

Min Cao[1] *   Shiqun Zhu[1] †

[1] *School of Physical Science and Technology,Suzhou University,*
Suzhou, Jiangsu 215006, People's Republic of China

Abstract. A general scheme for teleporting an M-particle d-level quantum state is proposed, when M-pair of partially entangled particles are utilized as quantum channels, independent general Bell state measurements are performed for joint measurement. Explicit expressions of two unitary reductions the receiver adopts to reconstruct the original quantum state of the sender are derived. Quantum teleportation can be successfully realized with a certain probability, which is determined by the smallest coefficients of each entangled pairs. It is shown that when all the quantum channels are maximally entangled the total probability equals 1.

Keywords: quantum entanglement, quantum teleportation

As one of the most important fields of quantum information theory, Quantum teleportation has been paid much attention both theoretically and experimentally due to its exciting applications in quantum communication and quantum computation. Since the original scheme proposed by Bennett et al.[1], quantum teleportation has been especially interested in single-particle of two-level, N/d-dimensional, and continuous variable states [1∼4]. Recently, schemes for teleportation of multi-particle two-level states are discussed [5∼7]. Conclusive teleportation of multi-particle d-level quantum states as a most important source of "one to many" and "many-to-many" communication protocol has been studied [8, 9].

In this Letter, a general scheme of probabilistic teleportation of an M-particle d-level quantum state is investigated, where M-pair of partially entangled particles are served as quantum information channel, independent Bell state measurements are performed on particles owned by Alice. Explicit expressions of two unitary operations corresponding to different Bell-basis measurements are deduced. It is showed that the probabilistic teleportation can be achieved with a successful probability which determined by the smallest coefficients of each entangled channels.

We should mention that in our scheme using general Bell-basis measurements and classical communication one can not teleport an unknown state with unit probability when the channel is non-maximally entangled. However, In Ref. [8] quantum state of one d-level particle is teleported via an multi-particle entangled state from one sender to M receivers. In Ref. [9] teleportation of N-particle d-level quantum state is performed via a maximally entangled (N+M)-particle state from N senders to M receivers. In their schemes the maximally entangled particles shared by Alice and Bob function as a quantum channel distribute for the faithful transmission.

We present our general scheme. For an M-particle d-level system, the initial unknown state we wish to teleport is

$$|\psi\rangle = \sum_{k_1k_2\cdots k_M=0}^{d-1} x_{k_1k_2\cdots k_M}\left|k_1k_2\cdots k_M\right\rangle \tag{1}$$

where $\sum\left|x_{k_1k_2\cdots k_M}\right|^2 = 1$ and $\{|k_1k_2\cdots k_M\rangle\}$ is the computational basis. To teleport the state $|\psi\rangle$, Alice (sender) needs to set up M-pair of quantum channels between her and Bob (receiver), which are partially entangled states with unknown amplitude C_a^N $(a=0,1,\cdots d-1, N=1,2,\cdots M)$

$$|\varphi\rangle = \sum_a C_a^N \left|aa\right\rangle_{(M+2N-1)(M+2N)} \tag{2}$$

Generally speaking, all the coefficients are different with $C_\alpha^N \neq C_\beta^N$. Without losing generality, it is assumed that $C_0^N = \min\left\{C_a^N\right\}$. Particles $(M+1), (M+3), \cdots, (3M-1)$ belong to Alice, particles $(M+2), (M+4), \cdots, 3M$ belong to Bob.

The state of system is $|\Psi\rangle = |\psi\rangle\,|\varphi\rangle_{(M+1)(M+2)}\,|\varphi\rangle_{(M+3)(M+4)}\cdots|\varphi\rangle_{(3M-1)(3M)}$ at this time. In order to realize the teleportation, Alice operates a series independent Bell-basis measurements on her particles.

$$\left|\phi_{nm^{(N-1)}}\right\rangle = \frac{1}{\sqrt{d}}\sum_j e^{2\pi ijn/d}\left|j\right\rangle_N \left|j, m^{(N-1)}\right\rangle_{M+2N-1} \tag{3}$$

where we denote $j, m^{(N-1)} = (j+m^{(N-1)}) \bmod d$, and $j, n, m^{(N-1)} = 0, 1, \cdots d-1$. After Alice completes measurements on her particles, she informs Bob the value N, n and $m^{(N-1)}$. To reconstruct the initial state, two unitary transformations will be applied. First Bob operate relevant unitary transformation U against different measurement result of Alice.

$$U = \bigotimes_{N=1}^{M} U_{nm^{(N-1)}}^{M+2N} \tag{4}$$

where $U_{nm^{(N-1)}}^{M+2N}$ take the form

$$U_{nm^{(N-1)}}^{M+2N} = \sum_k e^{2\pi ikn/d}\left|k\right\rangle_{M+2N}\ {}_{M+2N}\!\left\langle k, m^{(N-1)}\right| \tag{5}$$

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*caomin@suda.edu.cn

†szhu@suda.edu.cn

which change particles $(M+2),(M+4),\cdots,3M$ of Bob into

$$\left|\widehat{\psi}\right\rangle = \frac{1}{d^{M/2}}\sum x_{k_1k_2\cdots k_M}\left(\prod_N C^N_{k_N,m^{(N-1)}}\right)|k_1k_2\cdots k_M\rangle \tag{6}$$

Then by introducing an auxiliary two-level particle A with the initial state $|0\rangle$, one performs a collective unitary transformation on particles $(M+2),(M+4),\cdots,3M$ and A [11]. Under the basis $\{|k_1k_2\cdots k_M\rangle|0\rangle,|k_1k_2\cdots k_M\rangle|1\rangle\}$ the unitary transformation V may take the form

$$V=\left[\begin{array}{cc} B_{mm'\cdots m^{(M-1)}} & D_{mm'\cdots m^{(M-1)}} \\ D_{mm'\cdots m^{(M-1)}} & -B_{mm'\cdots m^{(M-1)}} \end{array}\right] \tag{7}$$

where

$$B_{mm'\cdots m^{(M-1)}} = diag\left(b^{mm'\cdots m^{(M-1)}}_{00\cdots 0}, b^{mm'\cdots m^{(M-1)}}_{00\cdots 1},\right.$$
$$\left.\cdots, b^{mm'\cdots m^{(M-1)}}_{(d-1)(d-1)\cdots(d-1)}\right) \tag{8}$$

$$D_{mm'\cdots m^{(M-1)}} = diag\left(\sqrt{1-\left(b^{mm'\cdots m^{(M-1)}}_{00\cdots 0}\right)^2},\right.$$
$$\left.\sqrt{1-\left(b^{mm'\cdots m^{(M-1)}}_{00\cdots 1}\right)^2},\cdots,\sqrt{1-\left(b^{mm'\cdots m^{(M-1)}}_{(d-1)(d-1)\cdots(d-1)}\right)^2}\right) \tag{9}$$

with $b^{mm'\cdots m^{(M-1)}}_{k_1k_2\cdots k_N} = \prod C^N_0/\prod C^N_{k_N,m^{(N-1)}}$. It changes the state $\left|\widehat{\psi}\right\rangle_{(M+2)(M+4)\cdots 3M}|0\rangle_A$ into

$$\tfrac{\prod C^N_0}{d^{M/2}}\sum x_{k_1k_2\cdots k_M}|k_1k_2\cdots k_M\rangle|0\rangle + \tfrac{1}{d^{M/2}}\sum x_{k_1k_2\cdots k_M}$$
$$\sqrt{\prod C^N_{k_N,m^{(N-1)}} - \prod C^N_0}\,|k_1k_2\cdots k_M\rangle|1\rangle \tag{10}$$

Then Bob performs an orthogonal measurement on particle A. According Eq. (9), if the result is $|1\rangle_A$, the teleportation is failed. If the measurement result is $|0\rangle_A$, the state of Bob's particles collapse to

$$\frac{\prod C^N_0}{d^{M/2}}\sum x_{k_1k_2\cdots k_M}|k_1k_2\cdots k_M\rangle \tag{11}$$

which is the original state one want to transform. The teleportation is successfully realized, and the probability of successful teleportation is $\left(\prod C^N_0\right)^2/d^M$.

This scheme is actually to teleport particles one-by-one, each quantum channel transforms one particle. To teleport M-particle state it requires M pairs of partially entangled particles. It is a generalization of a class of teleportation schemes [5, 10, 11]. In these schemes, two unitary transformations are applied to reconstruct the unknown state on Bob's particles, which collapsed to the state depends on not only the parameters of the unknown state but also the partially entangled channels after Alice performs measurements. The teleportation is successful achieved with certain probability determined by the smallest coefficients of each channels. The total probability equals 1 when the channels are all maximally entangled.

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