Scheme for generating various types of light field entanglements by using beam splitters

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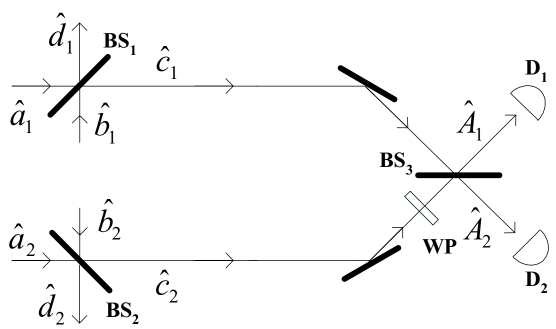
We propose an experimentally feasible scheme to generate various types of entangled states of light fields by using beam splitters and single-photon detectors. Two light fields are incident on two beam splitters and are split into strong and weak output modes respectively. A conditional joint measurement on both weak output modes may result in an entanglement between the two strong output modes. The conditions for the maximal entanglement are discussed based on the concurrence.

Keywords: Entangled State of Light Fields, Beam Splitter, Entanglement Concurrence

Quantum entanglement has been identified as a basic resource in achieving tasks of quantum communication and quantum computation [1]. Photons are considered to be the best quantum information carriers over long distances, and those in entangled states have been used to experimentally demonstrate quantum teleportation [2], quantum dense coding [3], quantum cryptography [4].

In comparison with other candidates for engineering quantum entanglement, light fields possess more abundant capacity to create various types of entangled states including the discrete, the continuous variable, and the combination of the both. Recently, it was found that a single beam splitter can act as an entangler for light fields if the input modes are in appropriate nonclassical states [5, 6]. However, the types of the resultant entangled states from the existent schemes are very limited. Very recently several schemes have been proposed to entangle distant atoms [7] and atomic ensembles [8] by means of photon interference. In this paper we extend such an idea to the generation of the entangled states of light fields.



The setup consists of three lossless beam splitters, BS$_1$, BS$_2$ and BS$_3$, and two single-photon detectors D$_1$ and D$_2$, as shown in Fig. 1. We assume that BS$_1$ and BS$_2$ are of the same amplitude reflection and transmission coefficients R and T with a relation $|R|^2 + |T|^2 = 1$. In our scheme BS$_j$ ($j = 1, 2$) has one non-zero input field in one input port, the other input port is always left in the vacuum state. The non-zero input field is split into two output fields. If R and T of the beam splitter BS$_j$ are largely different, one of the outputs will be very strong, and the other is very weak. In fact, we can design BS$_1$ and BS$_2$ in such a way that the weak output field possesses maximally one photon, that is, the weak output mode is in the vacuum state $|0\rangle$ or in the one photon number state $|1\rangle$. Correspondingly the strong output mode will be in a state that keeps the photon number of the input field unchanged or in a state that annihilates one photon from the input field since the lossless beam splitter conserves the photon number of the input fields. In this way we have prepared a specific entangled state of the weak and the strong output fields. Subsequently we let the two weak output fields from BS$_1$ and BS$_2$ be combined at BS$_3$, a 50%:50% beam splitter, and then detected by single-photon detectors D$_1$ and D$_2$. If only one photon is registered by D$_1$ or D$_2$, we successfully generate an entangled state of the two strong output fields of BS$_1$ and BS$_2$. Otherwise, we fail to generate the desired entangled state and should repeat the process again until one photon is registered.

In order to illustrate our method explicitly, let us denote by $\hat{a}_j, \hat{b}_j$ the input mode amplitudes shown in Fig. 1 and by $\hat{c}_j, \hat{d}_j$ the output mode amplitudes, where the subscript j stands for BS$_j$($j = 1, 2$). Suppose the initial quantum state of the input fields for BS$_j$ is a product state, $|0,\psi_j\rangle_{in} = |0\rangle_{a_j} \otimes |\psi_i\rangle_{b_j}$, in which the mode $\hat{a}_j$ is supposed to be always in the vacuum state $|0\rangle_{a_j}$ and the mode $\hat{b}_j$ in a superposition of the photon number states which can be expressed as

$$|\psi_j\rangle = \sum_{n=0}^{\infty} f_n^{(j)} |n\rangle, \tag{1}$$

In the case of the c_j mode has maximally one photon, the output fields are then of the following form [9],

$$|\Psi_j\rangle_{out} = |0\rangle_{c_j} \otimes |u_j\rangle_{d_j} + \exp(i\phi)\, R |1\rangle_{c_j} \otimes |v_j\rangle_{d_j}, \tag{2}$$

where $|u_j\rangle$ and $|v_j\rangle$ take the form

$$|u_j\rangle = \sum_{n=0}^{\infty} f_n^{(j)} T^n |n\rangle, \qquad (3a)$$

$$|v_j\rangle = \hat{d}_j \sum_{n=0}^{\infty} f_{n+1}^{(j)} T^n |n+1\rangle. \qquad (3b)$$

Now we show how to generate the entangled states of two strong output modes $\hat{d}_1$ and $\hat{d}_2$ by manipulating the two weak modes $\hat{c}_1$ and $\hat{c}_2$. As shown in Fig. 1, we suppose the two input fields are, respectively, incident on BS_1 and BS_2 simultaneously. Afterwards the two weak output modes $\hat{c}_1$ and $\hat{c}_2$ are combined at BS_3 with the output mode amplitudes $\hat{A}_1 = \left[i\hat{c}_1 + e^{i\gamma}\hat{c}_2\right]/\sqrt{2}$, $\hat{A}_2 = \left[\hat{c}_1 + ie^{i\gamma}\hat{c}_2\right]/\sqrt{2}$. Here γ denotes the phase shift induced by a wave plate (WP). The detection of a single photon by D_1 or D_2 is accompanied by the wave function collapse $|\Psi_1\rangle_{out} \otimes |\Psi_2\rangle_{out} \to \hat{A}_{1,2}(|\Psi_1\rangle_{out} \otimes |\Psi_2\rangle_{out})$. Neglecting the high-order terms $o(R^2)$, we find the final state of the two strong modes, conditional on a click of either D_1 or D_2, takes the following form,

$$|\Phi\rangle_{1,2} = |v_1\rangle|u_2\rangle \mp i\exp(i\gamma)\,|u_1\rangle|v_2\rangle. \qquad (4)$$

The probability for successfully generating the above state is proportional to R^2. The phase factor in state (4) can be controlled through the WP.

For the following analysis, we transform the state (4) to a normalized basis,

$$|\Phi\rangle_{1,2} = \nu_1\mu_2|V_1\rangle|U_2\rangle \mp i\exp(i\gamma)\,\mu_1\nu_2|U_1\rangle|V_2\rangle, \qquad (5)$$

where $|U_j\rangle = |u_j\rangle/\mu_j$, $|V_j\rangle = |v_j\rangle/\nu_j$ are normalized states with $\mu_j = \left(\sum_{n=0}^{\infty} |T^n f_n^{(j)}|^2\right)^{\frac{1}{2}}$ and $\nu_j = \left(\sum_{n=0}^{\infty} (n+1)\,|T^n f_{n+1}^{(j)}|^2\right)^{\frac{1}{2}}$.

The concurrence has been proved to be a convenient entanglement measure for such states and has been evaluated by Wang [10],

$$C_{1,2} = \frac{2\mu_1\nu_1\mu_2\nu_2\sqrt{(1-|\langle V_1|U_1\rangle|^2)(1-|\langle V_2|U_2\rangle|^2)}}{\mu_1^2\nu_2^2 + \nu_1^2\mu_2^2 + \mu_1\nu_1\mu_2\nu_2\left(\pm i e^{-i\gamma}\langle U_1|V_1\rangle\langle V_2|U_2\rangle \mp i e^{i\gamma}\langle V_1|U_1\rangle\langle U_2|V_2\rangle\right)}. \qquad (6)$$

The concurrence ranges from 0 to 1 with the value 1 corresponding to a maximally entangled state (MES). In a special case where the input field of BS_1 is the same as that of BS_2, *i.e.*, $f_n^{(1)} = f_n^{(2)}$, we have $|u_1\rangle = |u_2\rangle \equiv |u\rangle$, $|v_1\rangle = |v_2\rangle \equiv |v\rangle$, $\mu_1 = \mu_2$, $\nu_1 = \nu_2$, and therefore $|U_1\rangle = |U_2\rangle \equiv |U\rangle$, $|V_1\rangle = |V_2\rangle \equiv |V\rangle$, the concurrence is thus simplified as

$$C_{1,2} = \frac{1-|\langle V|U\rangle|^2}{1 \pm \sin\gamma|\langle V|U\rangle|^2}. \qquad (7)$$

From Eq. (7) one can readily deduce the following conclussion: (i) If $|U\rangle$ and $|V\rangle$ are orthogonal, i.e., $\langle V|U\rangle = \langle v|u\rangle = 0$, we always have $C_{1,2} = 1$, and the resultant entangled state (4) is an MES. (ii) If, however, $|U\rangle$ and $|V\rangle$ are nonorthogonal, the condition that state (4) is an MES is $\gamma = 3\pi/2$ when the photon is detected by D_1 or $\gamma = \pi/2$ when the photon is detected by D_2.

Using our scheme, the various types of entanglements of light fields, such as the discrete, the continuous variable light fields and the combination of the both, can be generated. The scheme is experimentally feasible because the basic elements in our scheme are accessible to experimental investigation with current technology.

[1] for a review see, e.g., C.H. Bennett, and D.P. DiVincenzo, Nature **404,** 247 (2000), and references therein.

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