Quantum Computation with Optical Coherent States

A. Gilchrist[1] * T. C. Ralph[1] G.J. Milburn[1] W.J. Munro[2] S. Glancy[3]

[1] *Centre for Quantum Computer Technology and Department of Physics, University of Queensland Brisbane, QLD 4072, Australia*
[2] *Hewlett Packard Laboratories, Filton Road Stoke Gifford, Bristol BS34 8QZ, U.K.*
[3] *Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA*

Abstract. We show that quantum computation circuits using coherent states as the logical qubits can be constructed from simple linear networks, conditional photon measurements and "small" coherent superposition resource states.

Keywords: Quantum Computation, Coherent States, Linear Optics

Quantum optics has proved a fertile field for experimental tests of quantum information science. However, quantum optics was not thought to provide a practical path to efficient and scalable quantum computation. This orthodoxy was challenged when Knill et al.[1] showed that, given single photon sources and single photon detectors, linear optics alone would suffice to implement efficient quantum computation. While this result is surprising, the complexity of the optical networks required is daunting.

More recently it has become clear that other, quite different versions of this paradigm are possible. In particular, by encoding the quantum information in multi-photon coherent states, rather than single photon states, an efficient scheme which is elegant in its simplicity has been proposed [2]. The required resource, which may be produced non-deterministically, is a superposition of coherent states (commonly referred to as "cat" states). Given this, the scheme is deterministic and requires only relatively simple linear optical networks and photon counting. Unfortunately the amplitude of the required resource states is prohibitively large. Here we build on this idea and show that with only a moderate increase in complexity a scheme based on much smaller superposition states is possible.

The power of the scheme stems from the ability to generate entangled states and make Bell basis measurements with simple linear interactions. This means teleportation protocols of various forms can be implemented deterministically to great effect. In particular, splitting a cat state of the form $1/\sqrt{2}(|-\sqrt{2}\ \alpha\rangle + |\sqrt{2}\ \alpha\rangle)$ on a 50:50 beamsplitter produces the entangled Bell state, $1/\sqrt{2}(|-\alpha,-\alpha\rangle + |\alpha,\alpha\rangle)$.

A synopsis of the basic elements of our scheme are as follows,

The qubits: the logical qubits are encoded in coherent states with $|0\rangle_L \equiv |-\alpha\rangle$ and $|1\rangle_L \equiv |\alpha\rangle$, where we take α to be real. The approximation of orthogonality is good for α even moderately large as $|\langle\alpha|-\alpha\rangle|^2 = e^{-4\alpha^2}$. We will assume that $\alpha \geq 2$, which gives $|\langle\alpha|-\alpha\rangle|^2 \leq 1.1 \times 10^{-7}$. Measurement of the qubit values can be achieved with high efficiency by homodyne detection with respect to a local oscillator phase reference.

*alexei@physics.uq.edu.au

Phase Rotation Gate, $R(Z,\theta)$: consider an arbitrary single qubit rotation about Z, $R(Z,\theta) = \exp\{-i\frac{\theta}{2}Z\}$. This can be implemented by displacing our qubit a small distance and then using teleportation (see Fig 1).

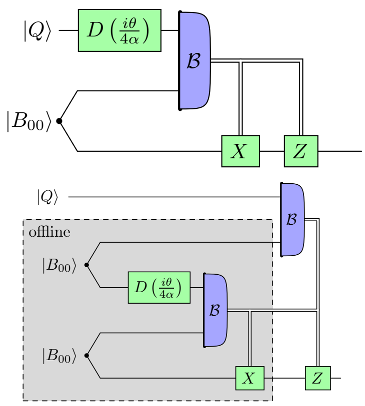

Figure 1: Schematics of implementing the $R(Z,\theta)$ gate. (a) The bare gate; its operation is near deterministic for a sufficiently small value of θ/α. Repeated application of this gate can build up a finite rotation with high probability. (b) The teleported gate; its operation is deterministic, however it may need to be applied several times in order to achieve the correct rotation. In the diagrams, $\mathcal{B}$ represents a cat-Bell measurement.

Controlled Phase Gate, $R(Z\otimes Z,-\phi)$: A non-trivial 2-qubit gate, $R(Z\otimes Z,-\phi) = \exp\{i\frac{\phi}{2}Z\otimes Z\}$, can be implemented in a similar way to the single qubit rotation (see Fig 2).

Superposition Gate, $R(X,\pi/2)$: to complete our set of gates we now describe how to implement a rotation of $\pi/2$ about X, ie $R(X,\pi/2) = \exp\{-i\frac{\pi}{4}X\}$. The gate is shown schematically in Fig 3.

The gates $R(Z,\theta)$, $R(X,\pi/2)$ and $R(Z\otimes Z,-\pi/2)$ form a universal set. An arbitrary single qubit rotation can be constructed from $R(Z,\psi)R(X,\pi/2)R(Z,\phi)R(X,-\pi/2)R(Z,\omega)$ and $R(Z\otimes Z,-\pi/2)$ is locally equivalent to a CNOT.

In order to implement the above gates we've made use of various simpler components such as X and Z gates,

(a) (b)

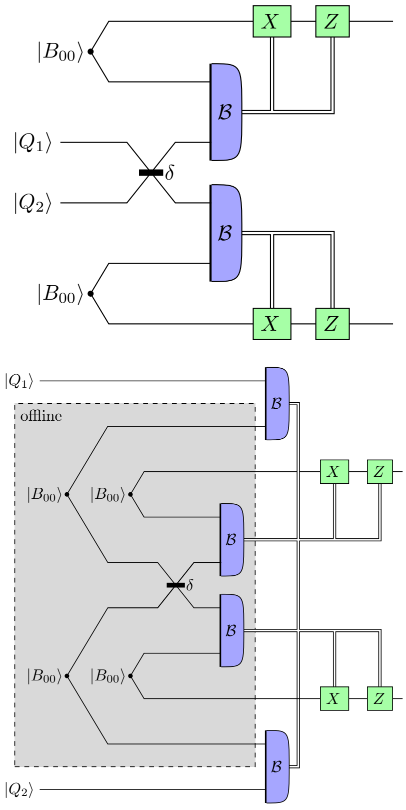

Figure 2: Schematics of implementing the $R(Z \otimes Z, -\pi/2)$ gate. (a) The bare gate; its operation is near deterministic for a sufficiently small value of $\theta^2\alpha^2$ where the reflectivity of the beamsplitter is $\delta = \cos^2 \frac{\theta}{2}$. Repeated application of this gate can build up to a $\pi/2$ rotation with high probability. (b) The teleported gate; its operation is deterministic.

and cat and cat-bell measurements. These can be implemented in the following manner

Bit-flip Gate X: the logical value of a qubit can be flipped by delaying it with respect to the local oscillator by half a cycle.

Sign-flip Gate Z: a "sign flip" or Z gate, can be achieved via teleportation [3] as follows. A Bell basis measurement is made on the qubit state and one half of the entangled Bell state. Depending on which of the four possible outcomes are found, bit flips may be necessary to correct the results. After X correction the gate has two possible outcomes: either the identity has been applied, in which case we repeat the process, or else the required transformation has been implemented.

Cat basis measurement, $\mathcal{C}$: we define a cat basis measurement to be some procedure that projects the state of an optical mode onto one of the two states $\frac{1}{\sqrt{2}}(|-\alpha\rangle \pm |\alpha\rangle)$. If our input state consists only of an arbitrary superposition of these 2 states then cat-basis measurement can be achieved by simply counting the photons in the mode. An even number of detected photons indicates measurement of the state $\frac{1}{\sqrt{2}}(|-\alpha\rangle + |\alpha\rangle)$, and an odd number of photons indicates measurement of $\frac{1}{\sqrt{2}}(|-\alpha\rangle - |\alpha\rangle)$. Of course, this will require very high quality photon detectors which can reliably distinguish n from $n+1$ photons when $n \sim \alpha^2$.

(a) (b)

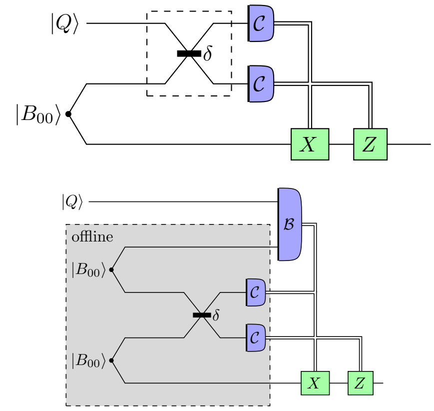

Figure 3: Schematics of implementing the $R(X, \pi/2)$ gate. (a) The bare gate; its operation is near deterministic for a sufficiently small value of $\theta^2\alpha^2$. Replacement of the dashed section with the repeated application of the gate of Fig.2(a) can build up to a $R(X, \pi/2)$ rotation with high probability. (b) The teleported gate; its operation is deterministic. In the diagrams, $\mathcal{C}$ represents a cat measurement.

Cat-Bell measurement, $\mathcal{B}$: in order to perform a Bell basis measurement on two modes (say, modes a and b) containing coherent state qubits we can employ the following procedure [4, 5]. Allow the two qubits to interfere at a 50:50 beam splitter, then use photon counters to measure the number of photons in each mode. We can then identify the four possible results: an even/odd number of photons in mode a and zero photons in mode b, or vice-versa. These results correspond to each of the four Bell-cat states.

In this presentation we will discuss the details and performance of this scheme.

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