Quantum error rejection code with spontaneous parametric down conversion

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Abstract. We propose a linear optical scheme to transmit an unknown qubit robustly over bit-flip-error channel. To avoid the technical difficulty of the standard quantum error correction code, our scheme is based on the concept of error-rejection. The whole scheme is based on currently existing technology. See details in quant-ph/0303010

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1 Bit-flip Channel:

Change the state $|u\rangle = \alpha|0\rangle + \beta|1\rangle$ into $|u_f\rangle = \alpha|1\rangle + \beta|0\rangle$ with a probability $\eta << 1/2$. After the channel, it is changed to

$$\rho_a = (1-\eta)|u\rangle\langle u| + \eta|u_f\rangle\langle u_f| \tag{1}$$

2 Encoding an unknown state:

$$|0\rangle \longrightarrow |00\rangle; |1\rangle \longrightarrow |11\rangle.$$

A CNOT operation on the qubit and the ancilla is enough to make the above encoding above.

3 Parity check:

Measure the parity of the code. Discard the code is we obtain 1 and decoding the code if we obtain 0.

4 Decoding

Measure the first qubit in X basis (basis of $|\pm\rangle$). If we obtain $|+\rangle$, the other qubit is in the original state; if we obtain $|-\rangle$ the other qubit is in the state of $\alpha|0\rangle - \beta|1\rangle$, we take a phase flip on state $|1\rangle$ and obtain the original state.

$$|u\rangle = \alpha|00\rangle + \beta|11\rangle = \frac{1}{\sqrt{2}}[|+\rangle|u\rangle + |-\rangle(\alpha|0\rangle - \beta|1\rangle)]$$

1. Initial state preparation. When one pair is emitted on each side of the nonlinear crystal, beam 0,1 and beam 2,3 are both in the entangled state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$. With the clicking of D0, the initial unknown state $|u\rangle = \cos(\gamma/2)|H\rangle + e^{i\phi}\sin(\gamma/2)|V\rangle$ is prepared on beam $1'$
2. Encoding. After step 1, the state of beam 1',2,3 is $|u\rangle_{1'}|\Phi\rangle_{23}$. With the combination of beam $1'$ and beam 2 by the PBS, the state for beam $2', 1'', 3$ is

$$\frac{1}{\sqrt{2}}\Big(\cos\frac{\gamma}{2}|H\rangle|H\rangle|H\rangle + e^{i\phi}\sin\frac{\gamma}{2}|V\rangle|V\rangle|V\rangle + \cos\frac{\gamma}{2}|0\rangle|HV\rangle|V\rangle + e^{i\phi}\sin\frac{\gamma}{2}|HV\rangle|0\rangle|H\rangle\Big). \tag{2}$$

Here the omitted subscripts are implied by $|w\rangle|s\rangle|t\rangle = |w\rangle_{2'}|s\rangle_{1''}|t\rangle_3$. We need only consider the first two terms above because other terms will never cause the required coincident. Rewritten the first two terms:

$$|+\rangle_{2'}\Big(\cos\frac{\gamma}{2}|HH\rangle_{1'',3} + e^{i\phi}\sin\frac{\gamma}{2}|VV\rangle_{1'',3}\Big) + |-\rangle_{2'}\Big(\cos\frac{\gamma}{2}|HH\rangle_{1'',3} - e^{i\phi}\sin\frac{\gamma}{2}|VV\rangle_{1'',3}\Big). \tag{3}$$

This shows that the state of beam $1'$ is indeed encoded onto beam $1''$ and beam 3 with the entangled state $\big(\cos(\gamma/2)|HH\rangle_{1'',3} + e^{i\phi}\sin(\gamma/2)|VV\rangle_{1'',3}|\big)$, if beam $2'$ is projected to single photon state $|+\rangle$.
3. Transmission through the bit flip channel. Beam $1''$ and beam 3 then each pass through a dashed line rectangular boxes which work as bit flip channels. We shall latter that how the rectangular box can work as the bit flip channel.
4. Parity check and decoding. After the code has passed through the noisy channel, one first take a parity check to decide whether to reject it or accept it. To do so one just observe beam $3''$. If it contains exactly 1 photon, the code is accepted otherwise it is rejected. Further, in decoding, one measures beam $3''$ in $|\pm\rangle$ basis (In our set-up this is done by first taking a Hadamard transformation to beam $3''$ and then measuring beam I3 in $|H\rangle, |V\rangle$ basis). If the no qubit in the code has been flipped after passing through the channel,the state for beam $1'''$ and beam $3'$ is $\big(\cos(\gamma/2)|HH\rangle_{1''',3'} + e^{i\phi}\sin(\gamma/2)|VV\rangle_{1''',3'}|\big)$ and this state keeps unchanged after passing through the PBS. Again the state of beam $3''$ and $I1$ can be rewritten into

$$|+\rangle_{3''}\Big(\cos\frac{\gamma}{2}|H\rangle_{I1} + e^{i\phi}\sin\frac{\gamma}{2}|V\rangle_{I1}\Big) + |-\rangle_{3''}\Big(\cos\frac{\gamma}{2}|H\rangle_{I1} - e^{i\phi}\sin\frac{\gamma}{2}|VV\rangle_{I1}\Big). \tag{4}$$

If beam $3''$ is projected to state $|+\rangle$, the original state is recovered in beam $I1$. Note that if one of the beam in $1'', 3$ is flipped, the polarization of beam $1'', 3'$ will be either H, V or V, H. This means beam $3''$ will be either in vacuum state or in the two photon state $|HV\rangle$. Beam I3 will be in the state $\frac{1}{\sqrt{2}}(|2H\rangle - |2V\rangle)$ given state $|HV\rangle$ for beam $3''$. In either case, beam $3''$ or beam y0 shall never contain exactly 1 photon. This shows that the code will be *rejected* with one qubit having been flipped. The code with both qubits having been flipped can also be accepted, but the probability of 2-flipping is in general

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very small. Therefore the error rate of all those states decoded from the *accepted* codes is greatly decreased.

5. Verification of the fault tolerance of QERC. To verify the fault tolerance property, we should observe the error rate of all the accepted qubits. The devices Pv-, RPBS, D1 and D4 are used to measure beam I1 in basis of $|u\rangle, |u^\perp\rangle = e^{-i\phi}\sin(\gamma/2)|H\rangle - \cos(\gamma/2)|V\rangle$. We shall only check the error rate to the *accepted* beams. For this we need check whether beam I0,x0 and y0 each contains exactly one photon in our scheme. The 4-fold clicking (D0,D2,D3,D1) or C_4 (D0,D2,D3,D4) guarantees this. For simplicity, we shall call the 4-fold clicking (D0,D2,D3,D1)as event C_1 and 4-fold clicking (D0,D2,D3,D4) as event C_4 hereafter. As we have shown, given the bit flip rate η, the average error rate without QERC is $E_0 = \eta/2$. The error rate for the accepted qubits with QERC is E_c. The experimental motivation is to observe the error rate withe QERC and to demonstrate this error rate is much less than E_0. The value E_c is obtained by the experiment. We shall count the error rate based on the number of each type of four fold events, i.e., C_1 and C_4. Denoting N_1, N_4 as the observed number of event C_1 and event C_4 respectively. The value $N_4/(N_1+N_4)$ is just the error rate for those accepted qubits with QERC.

The dashed boxes work as bit flip channels. For such a purpose, the phase shift $\theta(-\theta)$ or $\theta_1(-\theta_1)$ to vertical photon created by P1(-P1) or P1(-P1) should be randomly chosen from $\pm\frac{\pi}{2}$. The dashed box changes the incoming state to outgoing state by the following rule:

$$(|H\rangle,|V\rangle) \longrightarrow \sqrt{\frac{1}{1+\epsilon}}\left(|H\rangle+\sqrt{\epsilon}e^{i\theta}|V\rangle, |V\rangle-\sqrt{\epsilon}e^{-i\theta}|H\rangle\right) \quad (5)$$

Given state $|u\rangle$, after it passes a dashed square box, the state is changed to

$$|u_a\rangle = \sqrt{\frac{1}{1+\epsilon}}\left[|u\rangle - e^{i\theta}\sqrt{\epsilon}(\cos\frac{\gamma}{2}|V\rangle - e^{i\phi}e^{-2i\theta}\sin\frac{\gamma}{2}|H\rangle)\right] \quad (6)$$

Note that $e^{-2i\theta} = -1$, since θ is either $\frac{\pi}{2}$ or $-\frac{\pi}{2}$. Since $e^{i\theta}$ takes the value of $\pm i$ randomly, the state $|u_a\rangle$ is actually in an equal probabilistic mixture of both $\sqrt{\frac{1}{1+\epsilon}}\,[|u\rangle \pm i\sqrt{\epsilon}|u_f\rangle]$ therefore the output state of the dashed line square box is $\rho_a = \frac{1}{1+\epsilon}(|u\rangle\langle u| + \epsilon|u_f\rangle\langle u_f|)$. Here $|u_f\rangle$ is the bit-flipped state of $|u\rangle$. This shows that the flipping rate of the dashed box channel is

$$\eta = \frac{\epsilon}{1+\epsilon}. \quad (7)$$

Experimental set-up: Effects of imperfections of devices and source.

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5 References

References

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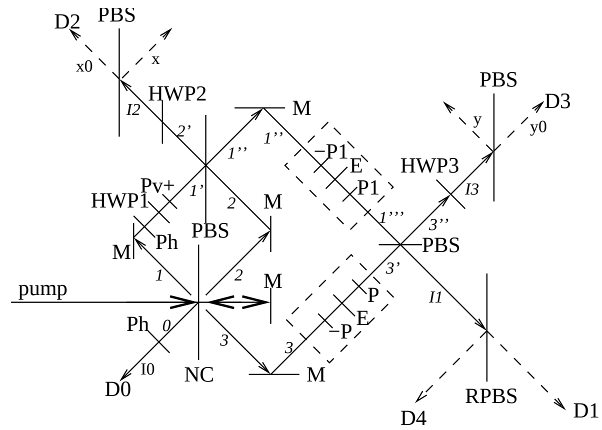

Figure 1: QERC with SPDC process. The rectangular boxes are bit-flip channels. Beam I1 is splitted by RPBS into $|u\rangle, |u^\perp\rangle$. NC: nonlinear crystal used in SPDC process. M: mirror. Ph: horizontal polarizer. HWP2 and HWP3: $\pi/4$ half wave plates. HWP1: $\gamma/2$ half wave plate. Pv+,Pv-: $\phi, -\phi$ phase shifters to vertically polarized photons only. PBS: polarizing beam splitter which transmits the horizontally polarized photons and reflects the vertically polarized photons. D0,D1,D2,D3,D4: photon detectors. P, -P, P1 and -P1: phase shifters, each of them takes a phase shift $\theta, -\theta, \theta_1, -\theta_1$ respectively to a vertically polarized photon only. θ, θ_1 each is a random value from $\pm\frac{\pi}{2}$. E: $\sin^{-1}\frac{\sqrt{\epsilon}}{\sqrt{1+\epsilon}}$ half wave plate.

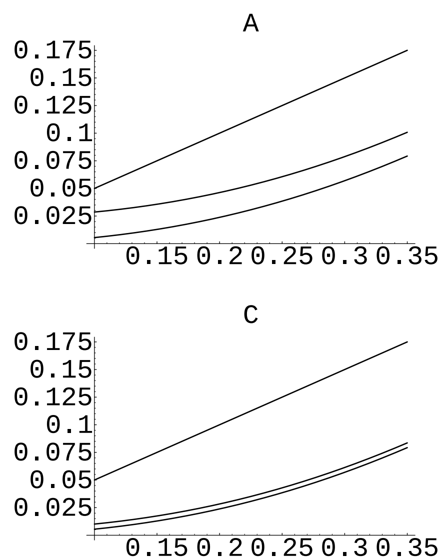

Figure 2: Expected effects of 3-pair emission and limitted detection efficiency. Horizontal axis: bit flipping rate η of the channel, Vertical axis: observed error rates after the qubits pass through the channel. The top straight line is for E_0. The lowest curve is for E_c, the curve upper to the lowest curve is for E_c', the upper bound of the observed values with imperfections. Fig. A,C are for the case of one pair emission probability $p = 1/100, 2/1000$ respectively in the SPDC process.