Measured Quantum Fourier Transform on Fiber optics

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Abstract. The Quantum Fourier transform (QFT) is a key function to realize quantum computations. A QFT followed by measurement (MQFT) was demonstrated on a simple circuit based on fiber-optics. Error probability was estimated to be 0.01 per qubit. The MQFT was shown to be robust against imperfections in the rotation gate. A successful MQFT of 255 qubits was performed with single-photon inputs.

Keywords: Quantum Fourier Transform, fiber-optics

1 Introduction

Shor's factorization algorithm [1] has proved the power of quantum computation over classical algorithms. The heart of the factorization and related quantum algorithms lies in the phase estimation algorithm [2]-[3] that consists of controlled-unitary operations and the quantum Fourier transform followed by a measurement (MQFT). The controlled-unitary operations provide an unknown phase according to the problem, and the MQFT then determines the phase to find the solution. The MQFT is known to be done semiclassically [4].

2 Quantum Circuit for MQFT

In the serial phase estimation [5], an eigenvalue of a unitary transform U defines the phase as $U|u\rangle=\exp[2\pi i\phi]|u\rangle$. Our task is to determine the phase expressed in n bits by $\phi=\phi_1 2^{-1}+\cdots+\phi_n 2^{-n}$. The circuit operates on the target qubits in an eigenstate of U with one control qubit at each step. Note that the initial control qubits are separable. The control qubits can be measured qubit by qubit, because they will be entangled with only the target qubits in the control unitary operation.

We implemented the MQFT circuit with fiber-optic devices. Qubits are represented by the polarization of a single photon. The key device of the circuit is the rotation gate that gives a relative phase shift to the $|1\rangle$ state. We employed a fiber loop with a phase modulator to implement the rotation gate. The loop configuration is often referred to as a Sagnac interferometer, where orthogonally polarized photons propagate in opposite directions through the same fiber. The two basis states are subject to the identical phase fluctuation, so that the present MQFT circuit is robust to disturbances.

The MQFT was demonstrated by putting a photon pulse sequence into the fiber-optic circuit. The photons were elliptically polarized according to random bit values $\phi_n,\cdots,\phi_1$. This simulated the output of the controlled-unitary operations. The average photon number in the pulse was set to less than one: 0.7 photon/pulse at the input of the circuit and 0.1 photon/pulse at the photon detector. The bit values were compared with the input bits. Figure 1 shows the results of 21 trials of 256 qubits. The inset shows the result of each trial. 'Successfully transformed bits' in the figure refers to the number of bits for which a QFT operation was done successfully. The distribution of the successful bits n obeyed a geometric distribution $E(n)=(1-p)^n p$ with the error probability per qubit p=0.01. We succeeded making a QFT of 256 qubits in two trials. The average of the successfully transformed bits was 97. Further statistical analysis showed that the error probability per qubit was in the range of $2.6\times10^{-3}\le p\le 1.2\times10^{-2}$ with the confidence level of 95%.

The errors originated from imperfections in the interferometer and in the phase modulation. Dark

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counts were negligibly small in the photon detector [6]. The performance of the interferometer is characterized by visibility, v=98%. The errors in the phase modulation result from the approximate rotation angle Φ_k and from the error in converting the rotation angle into the drive voltage. The latter can be reduced by careful calibration. The accuracy of the phase shift was determined by the precision of the electric pulse applied to the phase modulator. In most pulse generators, the precision is limited to three digits, i.e., 8-10 bits. This implies the rotation angle Φ_k should be truncated at the m-th bit (m<10). The phase error due to the truncation should not be significant [7], because the contribution from the j-th bit (j>m) decreases with the factor of $2^{-(j+1)}$. Truncation at the m-th bit results in the phase error

$$\delta = 2\pi\sum_{j=m}^{k-1}\frac{1}{2^{j+1}}\phi_{n-k+j+1} \le 2\pi\sum_{j=m}^{k-1}\frac{1}{2^{j+1}}$$
$$\le 2\pi\sum_{j=m}^{\infty}\frac{1}{2^{j+1}} = \frac{2\pi}{2^m} \qquad (1)$$

We measured the phase error in 24151 rotations for m=5. The values of cosd were distributed in [0.98,1] and [-1,-0.98], with a mean value of ±0.9936. The worst values (±0.98) corresponded to a phase error of $\pi/16$. The obtained values agree quite well with Eq. (1)'s prediction. v and δ determine the error probability of the measurement: $p=(1-v\cos\delta)/2$. The values $p=8.2\times10^{-3}$ (by using $\langle\cos\delta\rangle$=0.9936) and $p=1.5\times10^{-2}$ (by using $\cos\delta_{max}$=0.98) agree with those estimated from the error of the QFT trials.

3 Conclusion

We have shown that MQFT with decision by majority is simple to implement and robust. The fault-tolerant QFT reduces the size of quantum circuit required to perform phase estimation. The remaining problems are in realizing the controlled-unitary operation; the QFT by itself will not provide an exponential speed up in comparison with classical algorithms. One possible construction of a quantum computer is the combination of photon qubit and atomic qubits. The photon qubit serves as a control qubit, which switches the unitary transform in the quantum circuit of target qubits made by atoms. The photon probes the circuit as the change in the polarization. MQFT then extracts the eigenvalue of the unitary transform.

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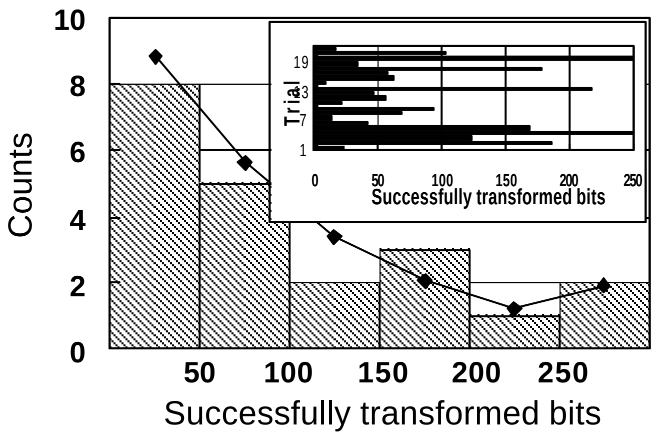

Fig. 1 MQFT results of 256 qubits. Bars represent the distribution of the successful bits obtained in the experiment. The line is for a geometric distribution with p=0.01. The inset shows the result of each trial.